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Jan 2-7:29 AM

How we got here (integration)
considered "area problem"
→ antiderivative is actually an "area under a curve" function
→ overlay the region with smaller and smaller width rectangles

idea: limit (width [of each rectangle]->0): area

Limits
Derivatives (differential calculus)
Integrals (integral calculus)
Infinite Series

theory and applications

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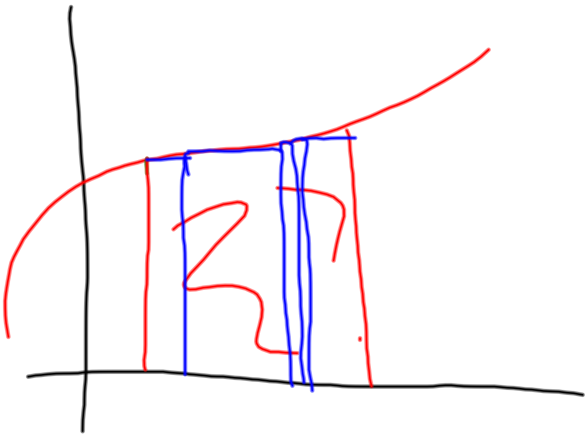
Problem we have considered only rectangles with identical widths



difficulty:
number of rectangles $\to \infty$
but there are still large chunks

Nuance:
we wish to choose,
as heights of rectangles,
f(# anywhere in that subinterval)

Nuance: a slight detail we need to be aware of

"Definition" A partition of a closed interval is a collection of "connected" subintervals whose union is the entire interval and any two of which intersect in a single point

A partition of $[0,2]$

a) $[0,2]$

b) $[0,1],[1,2]$

c) $[0,\frac{1}{2}],[\frac{1}{2},\frac{3}{2}],[\frac{3}{2},\frac{7}{4}],[\frac{7}{4},2]$

To get a limit . . .

definition: The mesh size of a partition is (simply) the width of the largest subinterval in the partition

A partition of $[0,2]$

a) $[0,2]$ mesh size = 2

b) $[0,1], [1,2]$ mesh size = 1

c) $[0,\frac{1}{2}], [\frac{1}{2},\frac{3}{2}], [\frac{3}{2},\frac{7}{4}], [\frac{7}{4},2]$ mesh size = 1

The area will be

$$\int_a^b f(x)\,dx \overset{\text{defined as}}{\equiv} \lim_{\substack{\text{mesh size} \\ \to 0}} \sum_{k=1}^{n} \Delta x_k \, f(x_k^*)$$

finite sum of n rectangles

an x-value in the k^{th} sub-int in the partition

called the $\boxed{\text{definite integral}}$

limit as mesh size goes to 0 which FORCES more and more rectangles

width of the k^{th} rectangle

height of the k^{th} rectangle

summation of products of 2 quantities one of which goes to 0

definite integral

$$\int_a^b f(x)\,dx$$

integral sign

upper limit of integration

integrand

necessary

lower limit of integration

rules of definite integrals

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_1^2 f(x)\,dx = -\int_2^1 f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

regardless of the order of a, b, c

basic results from limit

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b f(x)-g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Note

$$\int_a^b f(x)\,dx = \lim_{\substack{\text{mesh size}\\ \to 0}} \sum_{k=1}^{n} \Delta x_k f(x_k^*)$$

Note that the original interval does NOT appear on the right hand side (the limit). It is implicit in the partitions chosen

Consequences of definition of definite integral

If $f(x) \geq 0$ on $[a,b]$ then

$$\int_a^b f(x)\,dx \geq 0$$

If $f(x) \geq g(x)$ on $[a,b]$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

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$f(x)$ is said to be integrable on $[a,b]$

if $\int_a^b f(x)\,dx$ exists

[i.e. if that limit exists].

$f(x)$ continuous on $[a,b] \Rightarrow f(x)$ integrable on $[a,b]$.

the hierarchy differentiable $\Rightarrow$ continuous $\Rightarrow$ integrable

6.5/1-36

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Example 6 $\displaystyle\int_0^1 5-3\sqrt{1-x^2}\,dx$

$$=\int_0^1 5\,dx-3\int_0^1\sqrt{1-x^2}\,dx$$

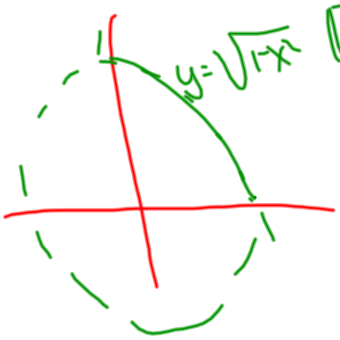
$\int_0^1 5\,dx$ is the area under $y=5$ from 0 to 1

which is $5*1=5$

$\int_0^1\sqrt{1-x^2}\,dx$ is the area in the first quadrant

of the region inside the unit circle

$y=\sqrt{1-x^2}$ [pythagorean Theorem]



$=\frac{1}{4}\left(\pi(1)^2\right)=\frac{\pi}{4}$

So answer is $5-3\left(\frac{\pi}{4}\right)$